Legendre Transforms and Maxwell's Equations

In general $\partial f = \left(\frac{\partial f}{\partial x}\right)_y \partial x + \left(\frac{\partial f}{\partial y}\right)_x \partial y$ for which the Euler Criteria is $\left(\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right)_y\right)_x = \left(\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)_x\right)_y$ we can derive almost everything we have done in class. Remember these two things are your pretty much set.

Starting from: $\partial U = \left(\frac{\partial U}{\partial S}\right)_V \partial S + \left(\frac{\partial U}{\partial V}\right)_S \partial V = T\partial S - P\partial V$ and using the Euler Criteria we get the first Maxwell equation: $\left(\frac{\partial}{\partial V}\left(\frac{\partial U}{\partial S}\right)_V\right)_S = \left(\frac{\partial}{\partial S}\left(\frac{\partial U}{\partial V}\right)_S\right)_V$ Plugging in the known variables $\left(\frac{\partial U}{\partial S}\right)_V = T$ and $\left(\frac{\partial U}{\partial V}\right)_S = -P$ gives us $\left(\frac{\partial T}{\partial V}\right)_S = \left(\frac{-\partial P}{\partial S}\right)_V$

Now for H which is U+PV. Now the Legendre transform gives us: $\partial H = T\partial S - P\partial V + P\partial V + V\partial P = T\partial S + V\partial P$ The first two terms are from δU and the last two terms are from taking the differential of +PV. This shows that H is a function of S and P, and we are set up to proceed as before.
Starting from: $\partial H = \left(\frac{\partial H}{\partial S}\right)_P \partial S + \left(\frac{\partial H}{\partial P}\right)_S \partial P = T\partial S + V\partial P$ and using the Euler Criteria we get the second Maxwell equation: $\left(\frac{\partial}{\partial P}\left(\frac{\partial H}{\partial S}\right)_P\right)_S = \left(\frac{\partial}{\partial S}\left(\frac{\partial H}{\partial P}\right)_S\right)_P$ Plugging in the known variables $\left(\frac{\partial H}{\partial S}\right)_P = T$ and $\left(\frac{\partial H}{\partial P}\right)_S = V$ gives us $\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P$

Now for A which is U-TS. The Legendre transform gives us: $\partial A = T\partial S - P\partial V - T\partial S - S\partial T = -S\partial T - P\partial V$. The first two terms are from δU and the last two terms are from taking the differential of -TS. This shows that A is a function of T and V, and we are set up to proceed as before.
Starting from: $\partial A = \left(\frac{\partial A}{\partial T}\right)_V \partial T + \left(\frac{\partial A}{\partial V}\right)_T \partial V = -S\partial T - P\partial V$ and using the Euler Criteria we get the third Maxwell equation: $\left(\frac{\partial}{\partial V}\left(\frac{\partial A}{\partial T}\right)_V\right)_T = \left(\frac{\partial}{\partial T}\left(\frac{\partial A}{\partial V}\right)_T\right)_V$ Plugging in the known variables $\left(\frac{\partial A}{\partial T}\right)_V = -S$ and $\left(\frac{\partial A}{\partial V}\right)_T = -P$ gives us $\left(\frac{-\partial S}{\partial V}\right)_T = \left(\frac{-\partial P}{\partial T}\right)_V$ which is also $\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$

Now for G which is U-TS+PV, or H-TS, or A+PV. You should now in your notes do the Legendre transform to get out last Maxwell equation, which is:
$\left(\frac{-\partial S}{\partial P}\right)_T = \left(\frac{\partial V}{\partial T}\right)_P$