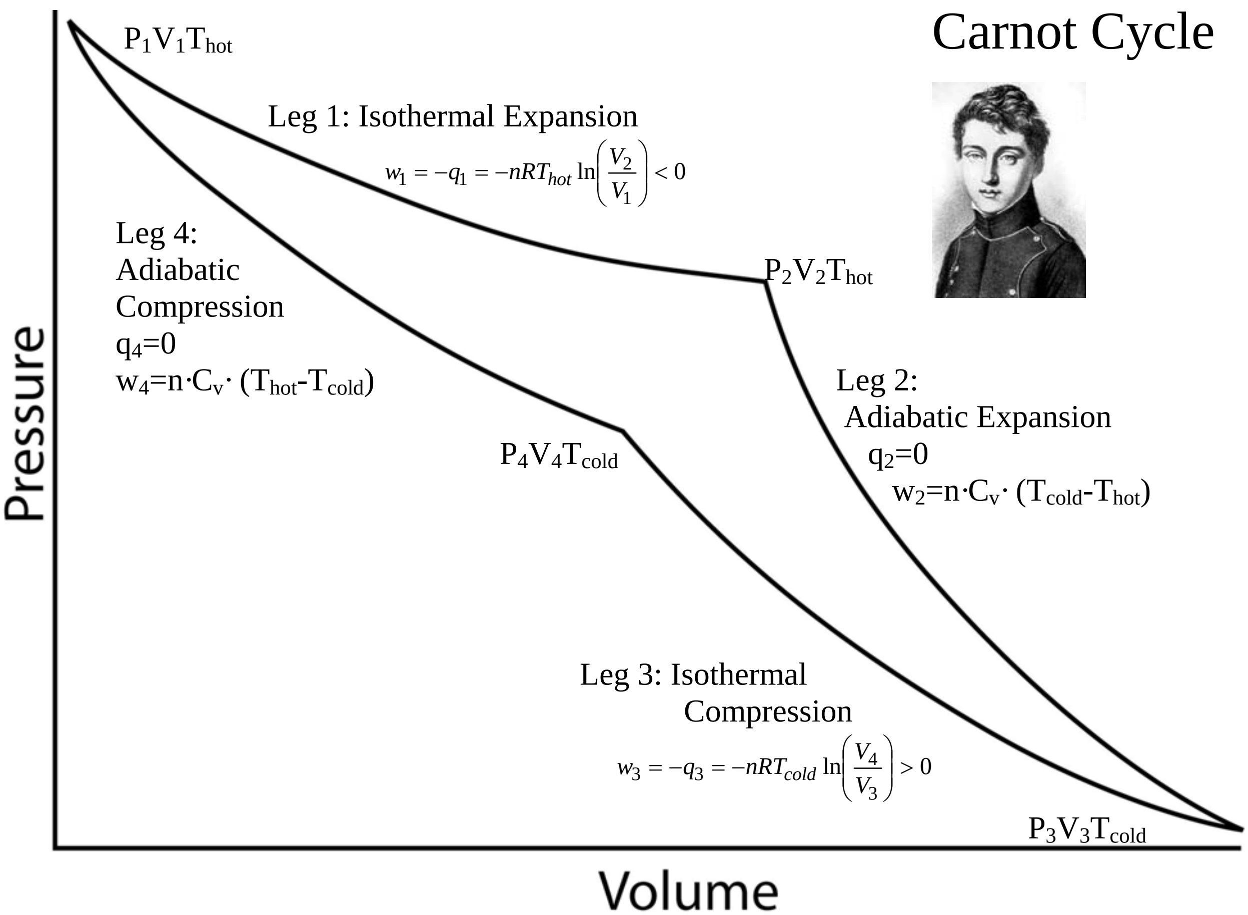

Carnot Cycle:

Work always adds (let it be negative or positive on its own) so the total work is:

$w_{tot}= w_1+w_2+w_3+w_4= -nRT_{hot}\ln\left(\frac{V_2}{V_1}\right) + n\cdot C_v\cdot (T_{cold}-T_{hot}) + -nRT_{cold}\ln\left(\frac{V_4}{V_3}\right) + n\cdot C_v\cdot (T_{hot}-T_{cold})$

Since $(T_{hot}-T_{cold})= -(T_{cold}-T_{hot})$ the above reduces to:

$w_{tot} = -nRT_{hot}\ln\left(\frac{V_2}{V_1}\right) - nRT_{cold}\ln\left(\frac{V_4}{V_3}\right)$ which is equivalent to $w_{tot}= -nRT_{hot}\ln\left(\frac{V_2}{V_1}\right) + nRT_{cold}\ln\left(\frac{V_3}{V_4}\right)$

where I used the identity: $\ln\left(\frac{V_4}{V_3}\right) = -\ln\left(\frac{V_3}{V_4}\right)$

Since we have 4 volumes but two temperatures we can relate the two via the adiabatic expansion and compression by relationships as:

$\left(\frac{T_f}{T_i}\right)^{\frac{C_v}{nR}} = \left(\frac{V_i}{V_f}\right)$ In our case: $\left(\frac{T_{cold}}{T_{hot}}\right)^{\frac{C_v}{nR}} = \left(\frac{V_2}{V_3}\right)$ and $\left(\frac{T_{hot}}{T_{cold}}\right)^{\frac{C_v}{nR}} = \left(\frac{V_4}{V_1}\right)$ which rearranges to:

$\left(\frac{T_{cold}}{T_{hot}}\right)^{\frac{C_v}{nR}} = \left(\frac{V_1}{V_4}\right)$ This makes $\left(\frac{V_1}{V_4}\right) = \left(\frac{V_2}{V_3}\right)$ which we can rearrange to $\left(\frac{V_3}{V_4}\right) = \left(\frac{V_2}{V_1}\right)$.

Again the total work is:

$$w_{tot} = -nRT_{hot}\ln\left(\frac{V_2}{V_1}\right) + nRT_{cold}\ln\left(\frac{V_3}{V_4}\right)$$

Which using the above identity reduces to:

$$w_{tot} = -nRT_{hot}\ln\left(\frac{V_2}{V_1}\right) + nRT_{cold}\ln\left(\frac{V_2}{V_1}\right)$$

$$w_{tot} = -nR\ln\left(\frac{V_2}{V_1}\right)\cdot(T_{hot} - T_{cold})$$

Lets see here, the change in temperature is positive, the natural log is positive, n and R are positive, so the negative of a bunch of positive numbers multiplied is negative. Thus, the Carnot Engine provides negative work only.

Now we can evaluate the relationship between the maximum work done and the input energy. That is and only is q_1, since q_2 (opening the hot gases to the tailpipe) doesn't require me as a person, taxpayer, and citizen to do anything (doing nothing on my way to nowhere in my SUV is how you can tell I am an American). So that leaves us with the efficiency of turning q to w as:

$$\frac{|w_{tot}|}{q_1} = \frac{nR\ln\left(\frac{V_2}{V_1}\right)\cdot(T_{hot} - T_{cold})}{nRT_{hot}\ln\left(\frac{V_2}{V_1}\right)} = \frac{(T_{hot} - T_{cold})}{T_{hot}} = 1 - \frac{T_{cold}}{T_{hot}}$$