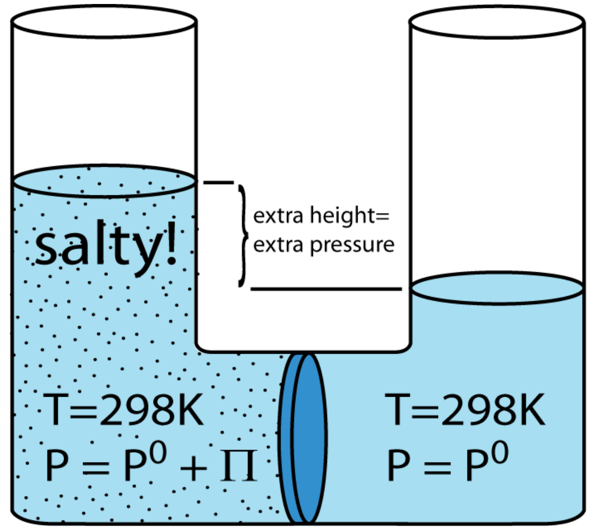
Osmotic Pressure!

Sorry to go so slow in class- I thought I would make it up with a handout on osmotic pressure.

Here is the setup:



You can see that the pure water on the left rushes into the liquid on the right to dilute the salt. However, the salt gains some height in the column which pushes back via pressure. The solution to the osmotic pressure is to realize that the chemical potential of the salt water on the left is equal to the pure liquid on the right due to the extra pressure.

First, note that the chemical potentials are equal:

$$\mu_{high\ pressure\ salt\ water} = \mu_{pure\ water} \qquad (1)$$

The chemical potential of the left is perturbed by the salt via the normal way:

$$\mu_{salt\ water} = \mu_{pure\ water} + RT \cdot ln(\chi_{water}) \ (2)$$

and pressure does this:

$$\mu_{pressurized\ water} = \mu_{pure\ water} + \int_{P^o}^{P^o+\Pi} V_m \partial P = \mu_{pure\ water} + V_m \cdot (P^o + \Pi - P^o)$$

$$= \mu_{pure\ water} + \Pi \cdot V_m \qquad (3)$$

where Π is the osmotic pressure. Note that there is normally a $-S_m \partial T$ term, but the left and right half of the liquid container are at the same temperature so this is 0 J/mol. Now put (1), (2) and (3) together:

$$\mu_{high\ pressure\ salt\ water} = \mu_{pure\ water} + \Pi \cdot V_m + RT \cdot ln(\chi_{water}) = \mu_{pure\ water}$$

Looking at the last two parts, we can see that the $\mu_{pure\ water}$ terms cancel which leaves:

$$\Pi \frac{V}{n_{water}} = -RT \cdot ln(\chi_{water}) = -RT \cdot ln(1 - \chi_{salt}) \cong RT \cdot \chi_{salt} \cong RT \cdot \frac{n_{salt}}{n_{water}}$$

where we have used the approximation of $\chi_{salt} = \frac{n_{salt}}{n_{salt}+n_{water}} \cong \frac{n_{salt}}{n_{water}}$, the fact that $ln(1 - \chi_{salt}) \cong -\chi_{salt}$, and the definitions of volume per mole $V_m = \frac{V}{n_{water}}$.

Last: $\Pi \frac{V}{n_{water}} \cong RT \cdot \frac{n_{salt}}{n_{water}}$

Which leaves the result: $\Pi \cdot V \cong n_{salt}RT$, note how this is basically PV=nRT except it's a liquid!