Adiabatic Changes of State

Starting with:

$$\mathrm{C_v\,\partial T = -P\,\partial V}$$

Since PV=nRT and thus $\mathrm{P = \frac{nRT}{V}}$, we find that the above changes to:

$$\mathrm{\frac{C_v\,\partial T}{T} = \frac{-nR\,\partial V}{V}}$$

Now we integrate:

$$\mathrm{\int_{T_i}^{T_f} \frac{C_v}{T}\,\partial T = \int_{V_i}^{V_f} \frac{-nR}{V}\,\partial V}$$

The integration yields:

$$\mathrm{C_v \cdot \ln\left(\frac{T_f}{T_i}\right) = -nR \cdot \ln\left(\frac{V_f}{V_i}\right)}$$

Since $\mathrm{\ln\left(\frac{x}{y}\right) = -\ln\left(\frac{y}{x}\right)}$ and with a little rearrangement:

$$\mathrm{\frac{C_v}{nR} \cdot \ln\left(\frac{T_f}{T_i}\right) = \ln\left(\frac{V_i}{V_f}\right)}$$

Last identity is $\mathrm{a \cdot \ln(x) = \ln(x^a)}$

$$\mathrm{\ln\left[\left(\frac{T_f}{T_i}\right)^{\frac{C_v}{nR}}\right] = \ln\left(\frac{V_i}{V_f}\right)}$$

Taking the exponential of both sides gives:

$$\mathrm{\left(\frac{T_f}{T_i}\right)^{\frac{C_v}{nR}} = \left(\frac{V_i}{V_f}\right)} \qquad (1)$$

It’s easy to show that this is equal to $\mathrm{\left(\frac{T_f}{T_i}\right) = \left(\frac{V_i}{V_f}\right)^{\frac{nR}{C_v}}}$.

Let's get to pressure next; since $C_p - C_v = nR$:

$$\left(\frac{T_f}{T_i}\right)^{\frac{C_v}{C_p - C_v}} = \left(\frac{V_i}{V_f}\right)$$

One other identity: $a^x = b$ then $a = b^{1/x}$ and substitution leads to:

$$\left(\frac{T_f}{T_i}\right) = \left(\frac{V_i}{V_f}\right)^{\frac{C_p}{C_v} - 1}$$

Last, recognize that

$$\left(\frac{T_f}{T_i}\right) = \left(\frac{P_f V_f}{P_i V_i}\right)$$

Plug this in above and rearrange the volumes:

$$\left(\frac{P_f}{P_i}\right) = \left(\frac{V_i}{V_f}\right)\left(\frac{V_i}{V_f}\right)^{\frac{C_p}{C_v} - 1}$$

Remember that $x \cdot x^n = x^{n+1}$

$$\left(\frac{P_f}{P_i}\right) = \left(\frac{V_i}{V_f}\right)^{\frac{C_p}{C_v}}$$

Here are a few others that can also be derived using the identities given above:

$$\left(\frac{P_i}{P_f}\right) = \left(\frac{V_f}{V_i}\right)^{\frac{C_p}{C_v}} \qquad \left(\frac{P_i}{P_f}\right)^{\frac{C_v}{C_p}} = \left(\frac{V_f}{V_i}\right) \qquad \left(\frac{T_i}{T_f}\right) = \left(\frac{V_f}{V_i}\right)^{\frac{C_p}{C_v} - 1}$$